Introduction to Casimir effect

F. Gliozzi

DFT & INFN, Università di Torino

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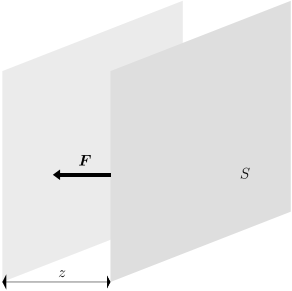

Plan of the talk

1. The Casimir Effect and the vacuum energy
2. The Casimir Effect from the Stefan-Boltzmann law
3. Generalisations
4. Main experiments
5. Applications
6. Conclusions

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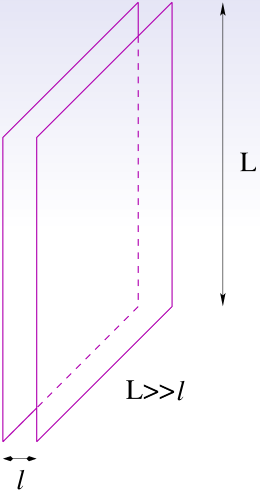
Casimir effect and the vacuum energy

The set-up

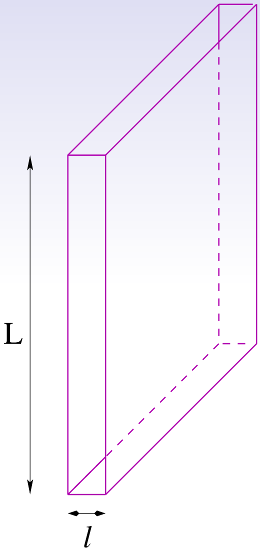


- Casimir , 1948:
- Two perfectly conducting, parallel, plates at a distance ℓ and at low temperature are subjected to an attractive force
- the pressure p depends only on ℓ

$$p = -\frac{\pi^2}{240}\frac{\hbar c}{\ell^4}$$

Why? Casimir explanation: zero-point energy

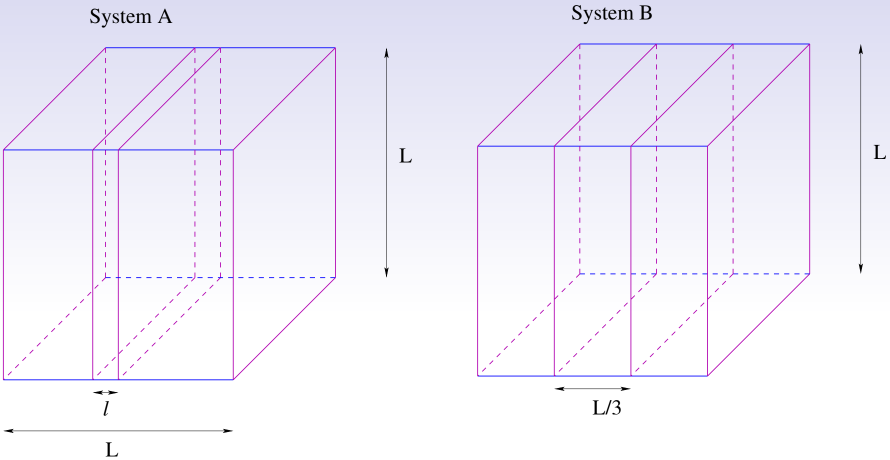
- The "empty" space is filled with an infinite family of decoupled harmonic oscillators associated to the normal modes of the electro-magnetic field
- The energy spectrum is $\hbar\omega(n + \frac{1}{2})$
- ω depend on the boundary conditions
- the number of allowed ω is infinite
- The vacuum zero-point energy $E_0 = \sum_{allowed\ \omega} \frac{1}{2}\hbar\omega$ *diverges*, however cannot be measured
- Only energy differences are measurable
- QFT puts $E_0 = 0$ in a large box
- it may be $E_0 \neq 0$ in a smaller system, like the region between two plates



- Fixed boundary conditions (conducting boundaries)
- $\omega = c\sqrt{(\frac{\pi m}{\ell})^2 + (\frac{\pi n}{L})^2 + (\frac{\pi k}{L})^2}$
  $n, m, k \geq 0$

➤ $$E_0 = 2 \sum_{n,m,k} \hbar\omega/2$$ diverges !

➩ Regularise by introducing a cut-off, e.g. $n, m, k < N$



$$E_0^A = E_0(\ell) + 2\,E_0(\tfrac{L}{2} - \tfrac{\ell}{2})$$

$$E_0^B = 3\,E_0(\tfrac{L}{3})$$

$$E_{Casimir} \equiv \lim_{L\to\infty} (\,E_0^A - E_0^B\,) \Rightarrow p = -\frac{\partial E_{Casimir}}{L^2\,\partial\,\ell}$$

A "judicious" choice of the cut-off yields

$$E_{Casimir} = -\hbar\, c \frac{\pi^2}{720} \frac{L^2}{\ell^3}$$

Agrees with other regularisations, but...

- P.A.M Dirac, 1980 :

  "... we should no longer have to make use of such illogical processes as infinite renormalisation. This is quite nonsense physically,and I have always opposed to it. It is just a rule of thumb that gives results"

Casimir effect as black body radiation in disguise

An intriguing relation- dimensional analysis

- Black body radiation
- Reflecting cavity in thermal equilibrium
- only one relevant scale: T

➩ $E_b/V = \frac{[E]}{[L^3]} = f(\kappa T, \hbar, c) = \sigma_b \frac{\kappa T}{(\hbar c/\kappa T)^3} = \sigma_b \frac{(\kappa T)^4}{(\hbar c)^3}$

- $\sigma_b = \frac{\pi^2}{15}$

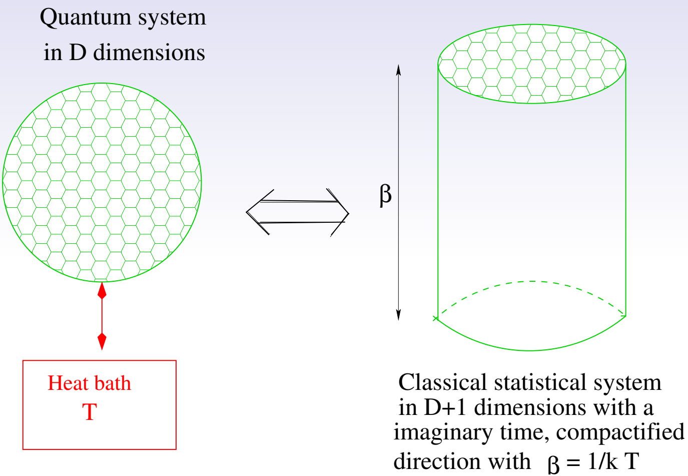
- Casimir effect
- Two parallel, conducting plates at $T \simeq 0$
- only one relevant scale : ℓ

➩ $E_c/V = \frac{[E]}{[L^3]} = g(\ell, \hbar, c) = \sigma_c \frac{\hbar c/\ell}{\ell^3} = \sigma_c \frac{\hbar c}{\ell^4}$

- $\sigma_c = -\frac{\pi^2}{720}$

$$\boxed{\sigma_b = -48\,\sigma_c}$$

Why?


Quantum system
in D dimensions
Heat bath
T
β
Classical statistical system
in D+1 dimensions with a
imaginary time, compactified
direction with β = 1/k T

Canonical partition function of a quantum system

$$Z \; = \; Tr\, e^{-\beta H} \; \equiv \sum_{\psi} \langle \psi |\, e^{-\beta H} \,| \psi \rangle \; , \quad \beta = 1/\kappa T$$

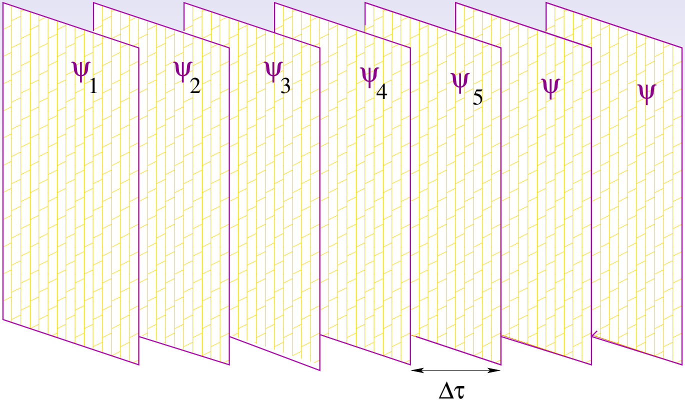
Put $\beta = N\Delta\tau \quad \Rightarrow$ Trotter formula:

$$Tr\, e^{-\beta H} = \lim_{N\to\infty} \sum_{\psi_1} \langle \psi_1 | (1 - \Delta\tau\, H) \sum_{\psi_2} |\psi_2\rangle \, \langle \psi_2 | (1 - \Delta\tau\, H)$$

$$\sum_{\psi_3} |\psi_3\rangle \, \langle \psi_3 | (1 - \Delta\tau\, H) \cdots \sum_{\psi_N} |\psi_N\rangle \, \langle \psi_N | (1 - \Delta\tau\, H) \, |\psi_1\rangle$$

$\Rightarrow \quad \beta$ new periodic dimension which acts as an imaginary time

Sum over configurations Ψ_i



Any quantum model in D dimensions at a temperature $T = 1/\beta$ is equivalent to a classical statistical system in $D + 1$ Euclidean dimensions. The temperature direction is periodic:

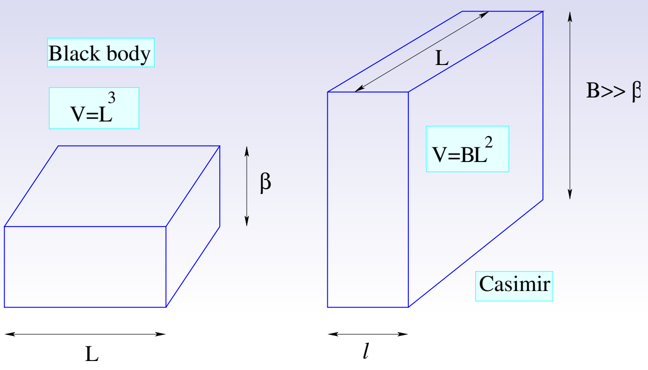
$$\psi(\vec{x}, \tau) = \psi(\vec{x}, \tau + \beta) \quad , \quad \vec{x} = (x_1, x_2, \cdots, x_D)$$

Black body radiation in equilibrium at a temperature T in a box $L \times L \times L$	$\Leftrightarrow$	Free electro-magnetic field in a 4-D box $\beta \times L \times L \times L$ periodic in β

- Configurations = stationary waves in a 4-D box
- eigenvalues of the 4-D laplacian ∂^2 [put $\hbar = c = \kappa = 1$]
  $\lambda_{n,m_i} = \left(\frac{2\pi n}{\beta}\right)^2 + \sum_{i=1}^{3} \left(\frac{\pi m_i}{L}\right)^2$
- $n = 0, \pm 1, \pm 2, \cdots, m_i = 1, 2, \ldots$
- Canonical partition function: $Z_b = 1/\sqrt{\det \partial^2}$
- No need of explicit calculations: the result is known from the elementary approach: $E_{black\ body} \equiv -\frac{\partial \log Z(\beta)}{\partial \beta} = \frac{\pi^2 V}{15 \beta^4}$

$\Downarrow$

$$\log Z_b = \frac{\pi^2 L^3}{45 \beta^3}$$



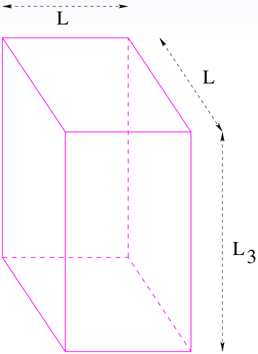
$$\log Z_b = \frac{\pi^2}{45}\frac{V}{\beta^3} = \frac{\pi^2}{45}\frac{L^3}{\beta^3}\ , \qquad \log Z_c = \frac{1}{2}\frac{\pi^2}{45}\frac{B\,L^2}{(2\ell)^3} = \frac{\pi^2}{720}\frac{B\,L^2}{\ell^3}$$

$$E_{black\ \ body} = -\frac{\partial \log Z_b}{\partial \beta} = \frac{\pi^2}{15}\frac{L^3}{\beta^4}\ ;\ E_{Casimir} = -\frac{\partial \log Z_c}{\partial B} = -\frac{\pi^2}{720}\frac{L^2}{\ell^3}$$

Generalisations

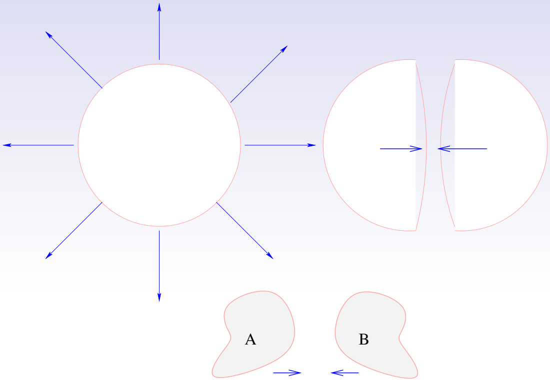
Variations on a theme: unstable boxes

- Generalisations to other space-time dimensions are straightforward
- many different shapes and boundary conditions have been studied
- the corresponding Casimir effect strongly depends on shapes and b.c.



- $0.408 < \frac{L_3}{L} < 3.48 \Rightarrow E_{Casimir} > 0$
- in particular in a conducting cube ($L_3 = L$)

$$E_{cube} \simeq 0.0916 \, \frac{\hbar c}{L}$$



- sphere: outward pressure Boyer,1968
- two hemispheres attract
- More generally, two bodies A,B related by a reflection attract Kenneth and Klich, 2006

Dynamical Casimir Effect

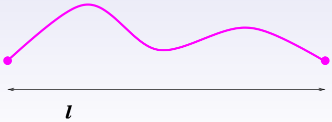
- If one of the plates is not fixed, but moves with a speed $\dot{\ell}$ there are perturbative corrections in $\dot{\ell}/c$ [Bordag, Dittes, Robaschik (1986)]

$$p = -\frac{\pi^2}{240}\frac{\hbar c}{\ell^4}\left[1 - \left(\frac{10}{\pi^2} - \frac{2}{3}\right)\frac{\dot{\ell}^2}{c^2} + O\left(\frac{\dot{\ell}^2}{c^4}\right)\right]$$

- If the mutual distance of the plates vibrates the system emits photons with a calculable rate [Dodonov and Klimov (1996)]

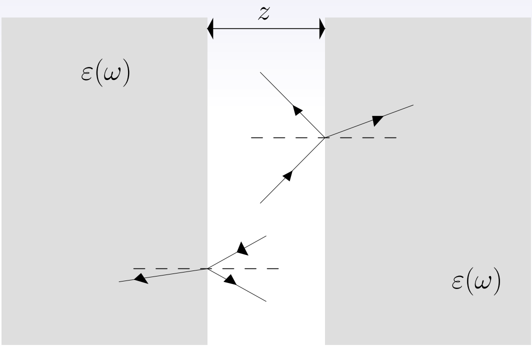
D=1 case: strings

- normal modes (fixed or free b.c.)
  $\omega = \frac{\pi}{\ell} n \ (n = 1, 2, \dots)$
- Spectrum: $\hbar\omega(m + \frac{1}{2})$
- Zero point energy
  $E_o = (d-2)\frac{\pi}{2\ell}\sum_{n=1}^{\infty} n$ **diverges !**



- $\zeta-$function regularisation [F.G.(1976), Hawking (1977)]:
  $\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}$ holomorphic function with a simple pole at $s = 1$
- $\zeta(-1) ='' \sum n''$ is **finite**, namely $\zeta(-1) = -\frac{1}{12}$
- $E_o = (d-2)\frac{\pi}{2\ell}\zeta(-1) = -(d-2)\frac{\pi}{\ell}\frac{1}{24}$
- This has an important role in the string theory

Toward realistic boundary conditions



Toward realistic boundary conditions

- Finite conductivity $[\epsilon < \infty]$ (Lifshitz 1956)

$$E_{Casimir}(\epsilon) = E_0 \left(\frac{\epsilon - 1}{\epsilon + 1}\right)^2 \phi(\epsilon)$$

- Free electron plasma in the metal (Bezerra, Klimchitskaya, Mostepanenko 2000)

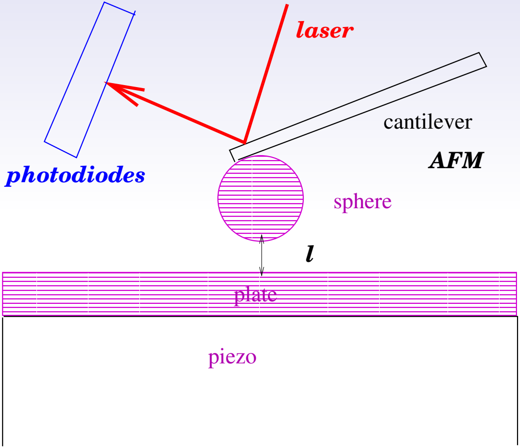
$$E_{Casimir}(\alpha) = E_0 \left[1 - \frac{16}{3}\alpha + 24\,\alpha^2 - \frac{640}{7}\left(1 - \frac{\pi^2}{210}\right)\alpha^3 + \cdots\right]$$

$$\alpha = \delta_o/\ell\ ,\ \delta_o = c/\omega_p\ ,\ \omega_p = \frac{4\pi N e^2}{m^*}$$

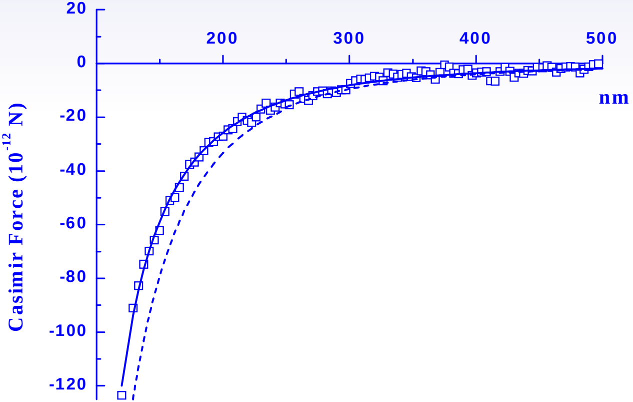
Experiments

- The Casimir force on a pair of plates of 1 cm^2 at a distance of 1 μm is $\sim 1.3 \times 10^{-7}\ N \Rightarrow$ a macroscopic effect, but extremely difficult to observe because of many parasitic effects
  - Van der Waals forces
  - roughness of surfaces
  - edge effects
  - hard to configure parallel plates at $\ell = 1\mu m$
  - finite conductivity of the plates
  - thermal fluctuations
  - residual electric charges on the surfaces
- First experimental observation : Tabor and Winterton, 1968
- truly convincing evidence with a laser interferometer: Lamoreaux, 1997

A precision experiment (Harris,Chen,Mohideen,2000)



- Plate-sphere config.
- Casimir force
  $F = -\frac{\pi^3 R}{360} \frac{\hbar c}{\ell^3}$
- polystyrene sphere coated with gold
  $R = 191\mu m$
- plate = optically polished sapphire disk (diameter 1 cm)
- ℓ is varied with the piezo-electric tube


20
0
-20
-40
-60
-80
-100
-120
200
300
400
500
nm
Casimir Force (10^-12 N)

Applications

Casimir forces and nano- systems

- At separations below a few ten nm the Casimir force dominates over other forces
- Movable components in nano-scales devices often stick together due to Casimir force (“stiction” process)
- This leads to poor yield in the fabrication of micro- and nano-mechanical systems
- It would be of much promise to develop systems with zero or suppressed Casimir force.

Casimir force as a test for new physics

- Many extensions of the standard model assume that the dimensionality of the space-time is larger than four
- The additional spatial dimensions are compactified at a small length scale
- some models suggest a length scale of the order of the fraction of millimeter
- As a result, the Newtonian gravitational potential is modified at short distances $V(r) = G\frac{m_1 m_2}{r} + \alpha\, e^{-r/\lambda}$ with $\lambda \sim 10\,\mu m$
- At $r \sim \lambda$ gravity is no longer the dominant force between neutral bodies. Casimir force is much stronger
- A precise determination of the Casimir force with the torsion pendulum is the best way to test the predictions of extra-dimensional physics

Conclusions

1. The Casimir effect is a quantum phenomenon which is strictly related to the nature of the vacuum
2. It strongly depends on the shape of the bodies involved and on the boundary conditions
3. It has been checked by precision experiments between metal surfaces
4. Its role in nano technology is rapidly increasing
5. It could be useful to test fundamental forces and new physics at the μm scale